

# Fuzzy Sets, Rough Sets, and Modeling Evidence: Theory and Application

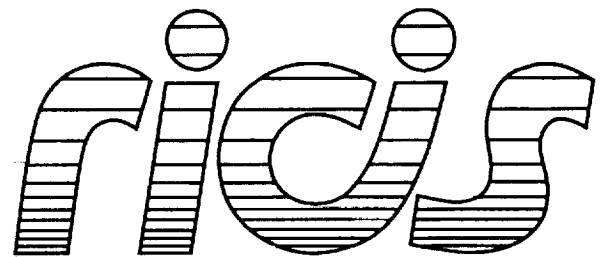
## *A Dempster-Shafer Based Approach to Compromise Decision Making with Multiattributes Applied to Product Selection*

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July 1992

Cooperative Agreement NCC 9-16  
Research Activity No. SR.01:  
Using Rough Sets for Knowledge Acquisition

NASA Johnson Space Center  
Information Systems Directorate  
Information Technology Division



Research Institute for Computing and Information Systems  
University of Houston-Clear Lake

(NASA-CR-190709) FUZZY SETS, ROUGH  
SETS, AND MODELING EVIDENCE: THEORY  
AND APPLICATION. A DEMPSTER-SHAFER  
BASED APPROACH TO COMPROMISE  
DECISION MAKING WITH  
MULTIATTRIBUTES APPLIED TO PRODUCT  
SELECTION (Research Inst. for  
Computing and Information Systems)  
31 p

N92-34149

Unclass

G3/61 0116937

TECHNICAL REPORT

IN-61-CIR

116937

R-31

## ***The RICIS Concept***

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The University of Houston-Clear Lake established the Research Institute for Computing and Information Systems (RICIS) in 1986 to encourage the NASA Johnson Space Center (JSC) and local industry to actively support research in the computing and information sciences. As part of this endeavor, UHCL proposed a partnership with JSC to jointly define and manage an integrated program of research in advanced data processing technology needed for JSC's main missions, including administrative, engineering and science responsibilities. JSC agreed and entered into a continuing cooperative agreement with UHCL beginning in May 1986, to jointly plan and execute such research through RICIS. Additionally, under Cooperative Agreement NCC 9-16, computing and educational facilities are shared by the two institutions to conduct the research.

The UHCL/RICIS mission is to conduct, coordinate, and disseminate research and professional level education in computing and information systems to serve the needs of the government, industry, community and academia. RICIS combines resources of UHCL and its gateway affiliates to research and develop materials, prototypes and publications on topics of mutual interest to its sponsors and researchers. Within UHCL, the mission is being implemented through interdisciplinary involvement of faculty and students from each of the four schools: Business and Public Administration, Education, Human Sciences and Humanities, and Natural and Applied Sciences. RICIS also collaborates with industry in a companion program. This program is focused on serving the research and advanced development needs of industry.

Moreover, UHCL established relationships with other universities and research organizations, having common research interests, to provide additional sources of expertise to conduct needed research. For example, UHCL has entered into a special partnership with Texas A&M University to help oversee RICIS research and education programs, while other research organizations are involved via the "gateway" concept.

A major role of RICIS then is to find the best match of sponsors, researchers and research objectives to advance knowledge in the computing and information sciences. RICIS, working jointly with its sponsors, advises on research needs, recommends principals for conducting the research, provides technical and administrative support to coordinate the research and integrates technical results into the goals of UHCL, NASA/JSC and industry.

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***A Dempster-Shafer Based Approach  
to Compromise Decision Making  
with Multiattributes Applied to  
Product Selection***



## **RICIS Preface**

**This research was conducted under auspices of the Research Institute for Computing and Information Systems by Dr. Andre' de Korvin of the University of Houston-Downtown. Dr. A. Glen Houston served as the RICIS research coordinator.**

**Funding was provided by the Information Systems Directorate, NASA/JSC through Cooperative Agreement NCC 9-16 between the NASA Johnson Space Center and the University of Houston-Clear Lake. The NASA research coordinator for this activity was Robert T. Savely of the Information Technology Division, NASA/JSC.**

**The views and conclusions contained in this report are those of the author and should not be interpreted as representative of the official policies, either express or implied, of UHCL, RICIS, NASA or the United States Government.**



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THEORY AND APPLICATION**

**A DEMPSTER-SHAFER BASED APPROACH TO  
COMPROMISE DECISION MAKING WITH MULTIATTRIBUTES  
APPLIED TO PRODUCT SELECTION**

**Report to RICIS  
Andre de Korvin**

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DECISION MAKING WITH MULTIATTRIBUTES  
APPLIED TO PRODUCT SELECTION

The Dempster-Shafer theory of evidence [7] is applied to a multiattribute decision making problem whereby the decision maker (DM) must compromise with available alternatives none of which exactly satisfies his ideal. The decision mechanism is constrained by the uncertainty inherent in the determination of the relative importance of each attribute element and the classification of existing alternatives. The classification of alternatives is addressed through expert evaluation of the degree to which each element is contained in each available alternative. The relative importance of each attribute element is determined through pairwise comparisons of the elements by the decision maker and implementation of a ratio scale quantification method. Then the Belief and Plausibility that an alternative will satisfy the decision maker's ideal are calculated and combined to rank order the available alternatives. Application to the problem of selecting computer software is given.



## I. INTRODUCTION

In this work we consider the problem of how to select a course of action when imperfect information is present. To make a decision, we look at designated attributes where each attribute has element values which may not be numerical. As an application, we will consider the problem of selecting software based upon the attributes of on-line help files (Help) and written documentation (Documentation); evaluating the elements of attribute Help as undesirable, acceptable, or desirable and the elements of Documentation as inadequate, adequate, and extensive.

Experts create a database where each alternative is classified relative to the amount by which each attribute element is present in each alternative. In our application, the degree to which the user/expert thinks a particular computer software package has an undesirable, acceptable, or desirable on-line help file is reflected in the assignment of weights to the elements; Undesirable Help, Acceptable Help, and Desirable Help. Every alternative will have such a classification for this attribute's and every other attribute's elements under consideration by the decision maker. This kind of classification reflects human uncertainty inherent in subjective judgments.

The uncertainty of subjective judgment is also present when a decision maker has to specify an optimal alternative.

The reason is that often an alternative is chosen by compromising according to the degree to which different attributes have distinct values. To determine this degree (or mass function) for each attribute, we determine the relative weight of importance of each attribute's elements. In our application, this is accomplished through the decision maker's pairwise comparisons of the elements of Help and Documentation, and the use of Guilford's ratio scale quantification process [5]. The optimal or ideal is formed by the relative weights for each attribute's elements combined over all attribute mass functions. For our application,  $\text{Ideal} = \text{Documentation} + \text{Help}$ .

To deal with the type of uncertainty present in the decision making situation described above, techniques other than classical logic need to be used. Although statistics may be the best tool available for handling likelihood, in many situations inaccuracies may result since probabilities must be estimated; sometimes without even the recourse to relative frequencies. The Dempster-Shafer theory of evidence [7] gives useful measures for the evaluation of subjective certainty. Fuzzy set theory is another tool used to deal with uncertainty where ambiguous terms are present. In the next section, we give the background information on fuzzy set theory and the Dempster-Shafer theory that is necessary to carry out our decision algorithm (see section IV) under the uncertainties pertaining to expert judgment and knowledge acquisition.

## II. BACKGROUND

Let  $X = \{x_1, x_2, \dots, x_n\}$ . The fuzzy subset of  $X$  is defined by a function from  $X$  into  $[0,1]$  ( $[2],[6],[10]$ ). That function is called the membership function. The notation  $\sum_i \alpha_i / x_i$  will refer to the fuzzy set whose membership function at  $x_i$  is  $\alpha_i$ . If  $A$  and  $B$  are fuzzy subsets of  $X$ , and if  $\mu_A$  and  $\mu_B$  are their membership functions then the membership functions of  $A \cap B$ ,  $A \cup B$  and  $\neg A$  are  $\mu_A \wedge \mu_B$ ,  $\mu_A \vee \mu_B$  and  $1 - \mu_A$ . This last expression denotes the fuzzy complement of  $A$ . (For additional details, see Zadeh [10].)

By a mass function on  $X$  we mean a function,  $m$ , that maps subsets of  $X$  into the reals with the properties:

$$(i) \quad m(\emptyset) = 0, \quad m(A) \geq 0$$

$$(ii) \quad \sum_{A \subset X} m(A) = 1$$

Subsets of  $X$  over which  $m$  is not zero are called focal elements of  $m$ . That is,  $U$  is a focal element of  $m$  if  $m(U) > 0$ . If  $m_1$  and  $m_2$  are two masses on  $X$ , then the direct sum of  $m_1$  and  $m_2$  is defined by  $(m_1 \oplus m_2)(A) = \sum_{B \cap C = A} m_1(B) m_2(C) / \sum_{B \cap C = \emptyset} m_1(B) m_2(C)$  if  $A \neq \emptyset$ . Here  $B$  and  $C$  denote (fuzzy) focal elements of  $m_1$  and  $m_2$ . Of course,  $A$  denotes a typical (fuzzy) focal element of  $m_1 \oplus m_2$ . Thus the focal elements of  $m_1 \oplus m_2$  are obtained by intersecting the focal elements of  $m_1$  and  $m_2$ . We set  $(m_1 \oplus m_2)(\emptyset) = 0$ . (For additional details, see Shafer [7].)

This rule of composition applies when  $m_1$  and  $m_2$  come from

independent sources of information and represent the mass generated by these two sources. The direct sum is a construct that sometimes models well the information gathered from independent sources of information, but this is not always the case. For a discussion of this, the reader is referred to the article by L.A. Zadeh [12]. In this context, the set  $X$  is often called the universe of discourse.

A mass function,  $m$ , on the universe of discourse,  $X$ , generates two important set functions defined on the sets of  $X$ . These are the belief and plausibility functions :  $Bel(B) = \sum_{A \subset B} m(A)$  and  $Pls(B) = \sum_{A \supset B} m(A)$ . The belief and plausibility functions denote a lower and an upper bound for an (unknown) probability function. For example, let  $S$  denote some area where oil may be present.

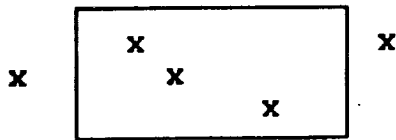


Figure 1:

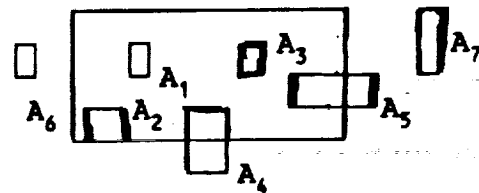


Figure 2:

In Figure 1, we have five experts locating points where oil could be found. Three of the five experts have located oil inside the area  $S$ , and two experts have located oil to be outside of  $S$ . We could say that the probability of oil inside  $S$  is  $3/5$ , since we have three hits out of five. In Figure 2 we have seven experts locating oil. These experts are not totally sure of themselves so the  $i^{th}$  expert locates the oil to be anywhere in  $A_i$  rather than at one specific point. Under these

circumstances, the probability of the oil being inside  $S$  is not defined, since the fourth and fifth expert are indicating that the oil might be inside or outside of  $S$ . If we seek the lowest possible probability that oil exists in  $S$ , we have 3 hits out of 7. If we want the highest probability we can say that we have 5 hits out of 7. The lowest probability is called belief and the highest probability is called plausibility.

If we define the focal elements of  $m$  to be  $\{A_1, A_2, \dots, A_7\}$  with  $m(A_i) = 1/6$ ;  $1 \leq i \leq 7$ . Then  $m_i$  is a mass function;  $\text{Bel}(S) = \sum_{A_i \subset S} m(A_i) = 3/7$  and  $\text{Pls}(S) = \sum_{A_i \cap S \neq \emptyset} m(A_i) = 5/7$ . It is clear that if the sets  $A_i$  are reduced to specific points, then Bel and Pls are equal and reduce to a probability function. Thus a probability can be viewed as a belief (or a plausibility) where the focal elements are points. The converse is not true; e.g. a belief function may not be viewed as a probability and the usual axioms for a probability function do not apply to a belief function. In fact, the formal axioms for a belief function are:

$$(i) \text{Bel}(\emptyset) = 0 \text{ and } \text{Bel}(X) = 1$$

For every collection of subsets,  $A_1, A_2, \dots, A_n$

$$(ii) \text{Bel}(A_1 \cup A_2 \cup \dots \cup A_n) \geq \sum_I (-1)^{|I|+1} \text{Bel}(\bigcap_{i \in I} A_i)$$

where  $I$  ranges over all non-empty finite subsets of  $\{1, 2, \dots, n\}$  and  $|I|$  denotes the cardinality of  $I$ . Any such function can be defined in terms of a mass  $m$  defined by  $m(A) = \sum_{B \subset A} (-1)^{|A-B|} \text{Bel}(B)$  where  $|A-B|$  is the cardinality of the set  $A \cap -B$ . Then  $\text{Bel}(B) = \sum_{A \subset B} m(A)$ .

A belief function is called Bayesian if

(i)  $\text{Bel}(\emptyset) = 0, \text{Bel}(X) = 1$

(ii)  $\text{Bel}(A \cup B) = \text{Bel}(A) + \text{Bel}(B)$

whenever A and B are disjoint. It may be shown that the following conditions are equivalent

(i) Bel is Bayesian

(ii) Focal elements are points

(iii)  $\text{Bel}(A) + \text{Bel}(\neg A) = 1$

In the present work, it will be very natural to extend this setting to the case where focal elements are fuzzy subsets of the universe of discourse. This setting was first considered by Zadeh [11], but Yager [8], among others, has done similar work. We would also refer the reader to a paper on the theory of masses over fuzzy sets by Yen [9], since we will define the different attributes and the ideal in those terms.

To begin the compromise decision making process, a set of alternatives  $h_1, h_2, \dots, h_t$  is defined. For our example, these will be different software packages under assessment for selection by the user. Let  $F_1, F_2, \dots, F_n$  denote a list of attributes considered to be important to the decision maker in the evaluation of the alternatives. For example,  $F_1$  could be Documentation,  $F_2$  could be Help, etc. Let  $f_i^{k_i}$  denote elements of the attribute,  $F_i$ , where  $1 \leq i \leq n$  and  $1 \leq k_i \leq n_i$ . For example, if  $i = 1$ ;  $f_1^1, f_1^2, f_1^3$  could denote Inadequate Documentation, Adequate Documentation, and Extensive

Documentation, respectively. Associated with each alternative,  $h_j$ , we have  $n$  fuzzy sets corresponding to the  $n$  different attributes. Thus,  $h_j$  is associated with  $\sum_{k_i=1}^{n_i} \alpha_{ij}^{k_i} / f_i^{k_i}$  where  $\alpha_{ij}^{k_i}$  is the amount to which element  $f_i^{k_i}$  of attribute  $F_i$  is present in alternative  $h_j$ ;  $1 \leq i \leq n$  and  $1 \leq j \leq t$ . For our example, the first computer package could be associated with .4/Inadequate Documentation + .5/Adequate Documentation + .1/Extensive Documentation + .3/Undesirable Help + .6/Acceptable Help + .1/Desirable Help, if we are concerned with the attributes, Documentation and Help.

We recognize that the decision maker may desire a particular element from the attributes under consideration more than another element. C.H. Coombs [4] proposed that there is such an ideal level of attributes for objects of choice. Using the notation for any alternative given above, we may express the ideal alternative as  $n$  fuzzy sets:

$\sum_{k_i=1}^{n_i} d_i^{k_i} / f_i^{k_i}$ ,  $1 \leq i \leq n$  and  $1 \leq k_i \leq n_i$  where  $d_i^{k_i}$  expresses to what degree the decision maker wants element  $f_i^{k_i}$  of  $F_i$ .

We may use a process such as Guilford's constant-sum method [5] to assist the DM in the evaluation process to determine his degree of preference for each  $f_i^{k_i}$  of  $F_i$ .

The assigning of relative weights of importance through pairwise comparisons of each attribute's elements asks the decision maker(s) to distribute a total of 100 points between the elements of each pair in the same proportion as the relative value of the two elements with respect to each other

[3]. After all of the comparisons have been made, the subjective values implicit in the decision maker's judgment are recovered through use of a ratio scale method ([3], [5]). The use of Guilford's [5] ratio scale method also allows the decision maker's consistency of judgment to be monitored [1]. It is necessary that the DM's ideal be as accurate as possible with respect to consistent weights of relative importance for each attribute's elements since these values form mass functions that ultimately influence the belief and plausibility of each alternative.

As Zeleny [14] suggests, the ideal serves as a minimum requirement for intelligent discourse. This ideal as generated by the relative importance weights of each attribute's elements reflects the decision maker's cultural, genetic, psychological, societal, and environmental background [13]. As a relatively unstable, context-dependent concept of informational importance, these weights are reflective of a given decision situation [13]. Thus, the relative importance values determined by the decision maker may vary for different sets of attribute elements, thereby altering the mass function associated with each attribute.

These mass functions contain focal elements which can be viewed as fuzzy subsets of alternatives. In other words, we can express each element of an attribute as a fuzzy set,  $F_i^{k_i}$ , of alternatives. For our example, we can write  $F_{\text{HELP}}^{\text{Undesirable}} = .3/h_1 + .5/h_2 + \dots + .8/h_t$  if  $h_1$ 's Help has been evaluated as .3



Undesirable,  $h_2$ 's Help has been evaluated as .5 Undesirable, etc. Using previous notation, we determine,  $F_i^{k_i}$  as  $\sum_{j=1}^t \alpha_{ij}^{k_i} / h_j$  where;  $1 \leq i \leq n$  and  $1 \leq k_i \leq n_i$ . Thus, associated with each element,  $f_i^{k_i}$  of each attribute,  $F_i$ , we can define fuzzy focal elements,  $F_i^{k_i}$ , over which mass functions can be determined. Indeed, we can define  $n$  masses,  $m_i$  ( $1 \leq i \leq n$ ) in terms of the ideal weight of each element,  $f_i^{k_i}$  for each attribute,  $F_i$  so that  $m_i(F_i^{k_i}) = d_i^{k_i}$ ;  $1 \leq i \leq n$  and  $1 \leq k_i \leq n_i$ . We define  $m$  by,  $m = m_1 \otimes \dots \otimes m_n$  where we use the combination rule [7], thereby forming mass function  $m$  over the intersection of finite sets of focal elements,  $F_i^{k_i}$ . We let  $A_\alpha$  be the fuzzy focal elements of  $m$ . For our example, we have nine fuzzy focal element sets,  $A_\alpha$ , formed from the two attributes Documentation and Help, each of which has three elements. (See section III). Using the definition of  $m_i$  we can determine the mass function,  $m$ , defined over the intersection,  $A_\alpha$ , of focal elements,  $F_i^{k_i}$  of  $m_i$ .

Following Zadeh's notation [11], we generalize the belief and plausibility function to:

$$\text{Bel}(B) = \sum_{\alpha} \inf (A_\alpha \rightarrow B) m(A_\alpha) \text{ and}$$

$$\text{Pls}(B) = \sum_{\alpha} \sup (B \wedge A_\alpha) m(A_\alpha).$$

and  $A_\alpha \rightarrow B$  is defined to be  $\neg A_\alpha \vee B$ . We now show that the definitions given are natural extensions of the crisp case.[11] We have  $(A_\alpha \rightarrow B)(x) = (\neg A_\alpha \vee B)(x)$

$$= \text{Max} (1 - A_\alpha(x), B(x))$$

$$\inf_x (A_a \rightarrow B) = \inf_x \text{Max}(1 - A_a(x), B(x))$$

When  $A_a$  and  $B$  are crisp sets:

$$A_a(x) = 1 \text{ if } x \in A_a; A_a(x) = 0 \text{ if } x \notin A_a$$

$$B(x) = 1 \text{ if } x \in B; B(x) = 0 \text{ if } x \notin B$$

Thus,

$\inf_x \text{Max}(1 - A_a(x), B(x)) = 1$  if and only if for all  $x$ ,  $A_a(x) = 0$  or  $B(x) = 1$ ;  $x \notin A_a$  or  $x \in B$ . This says that

$\inf_x \text{Max}(1 - A_a(x), B(x)) = 1$  if and only if  $A_a \subset B$ . Since the above expression can only be 0 or 1, in the crisp case,

$$\inf_x \text{Max}(1 - A_a(x), B(x)) = 0 \text{ if and only if } A_a \not\subset B.$$

Thus,

$$\inf_x (A_a \rightarrow B) = 1 \text{ if and only if } A_a \subset B; \text{ otherwise}$$

$$\inf_x (A_a \rightarrow B) = 0.$$

In the crisp case, the belief becomes  $\sum_{A_a \subset B} m(A_a)$  which coincides with the definition of  $\text{Bel}(B)$  given previously for the crisp case. Similarly, in the crisp case,

$\sup(B \wedge A_a) = \text{Max}_x \text{Min}(B(x), A_a(x)) = 1$  if and only if  $B(x) = 1$  and  $A_a(x) = 1$  for some  $x$ . That is, if  $x \in B$  and  $x \in A_a$  for some  $x$ . Thus, in the crisp case  $\sup(B \wedge A_a) = 1$  if and only if  $B \wedge A_a \neq \emptyset$ ; otherwise  $\sup(B \wedge A_a) = 0$ . The plausibility for the crisp case, then becomes  $\sum_{A_a \cap B \neq \emptyset} m(A_a)$  which coincides with the definition previously given for the crisp set.

We would like to specialize to the case where  $B = \{h_j\}$ . We have:

$$\text{Bel}(h_j) = \sum_a \inf (A_a \rightarrow h_j) m(A_a)$$

We begin by noting  $\inf_x (A_a \rightarrow h_j) = \inf_x \text{Max}(1 - A_a(x), h_j(x))$  where  $x$  ranges over  $\{h_1, h_2, \dots\}$ . Thus,  $h_j(x) = 1$  if  $x = h_j$  and  $h_j(x) = 0$  if  $x \neq h_j$ . Therefore,

$$\begin{aligned} \inf_x \text{Max}(1 - A_a(x), h_j(x)) &= \inf_{x \neq h_j} \text{Max}(1 - A_a(x), 0) \\ &= \inf_{x \neq h_j} (1 - A_a(x)) \\ &= 1 - \text{Max}_{x \neq h_j} A_a(x) \end{aligned}$$

$$\text{so } \inf (A_a \rightarrow h_j) = 1 - \text{Max}_{x \neq h_j} A_a(x)$$

$$\text{so } \text{Bel}(h_j) = \sum_a (1 - \text{Max}_{x \neq h_j} A_a(x)) m(A_a)$$

Similarly, it can be shown that the plausibility is given by:

$$\text{Pls}(h_j) = \sum_a A_a(h_j) m(A_a).$$

It should be noted that in crisp sets, plausibility is always greater than or equal to belief. Comparing the coefficients of the  $i^{\text{th}}$  terms for plausibility and belief, we have  $A_i(h_j)$  and  $1 - \text{Max}_{x \neq h_j} A_i(x)$ , respectively. Indeed, if  $A_i$  is a crisp set,  $1 - \text{Max}_{x \neq h_j} A_i(x) \in \{0, 1\}$ . It is equal to 1 if and only if  $\text{Max}_{x \neq h_j} A_i(x) = 0$  which implies that  $A_i = \{h_j\}$ . This, in turn, implies that  $h_j \in A_i$  which implies that  $A_i(h_j) = 1$ . In our research, we are not dealing with crisp sets, so plausibility is not necessarily greater than or equal to belief.

The gap between the plausibility and the belief of  $h_j$  represents the doubt about alternative  $h_j$ . If  $\text{Pls}(h_j)$  is high then the belief in the competing set is low since  $1 - \text{Pls}(h_j) = \text{Bel}(\neg h_j)$ . Hence, one way to select an alternative is to pick the alternative with the highest belief. Perhaps, a more sophisticated way is, in addition to the belief, compute

$\text{Bel}(-h_j)$ . It may be more desirable to pick an alternative whose belief is not a maximum when the belief in competing alternatives is low. In particular, if we consider the difference between the belief in an alternative and the belief in the competitors of that alternative:  $\text{Bel}(h_j) - \text{Bel}(-h_j) = \text{Bel}(h_j) - (1 - \text{Pls}(h_j))$ . Thus, the deciding factor for ranking the alternatives from highest to lowest could be  $\text{Bel}(h_j) + \text{Pls}(h_j)$ .

### III. EXAMPLE

The process developed in this paper allows the software user to actively participate as a decision maker in the selection of a set of packages by specifying a graded possibility distribution for each attribute that forms his ideal alternative. The Dempster-Shafer rule of combination of evidence evaluates information from independent alternatives to assess the degree of belief that each available package will satisfy the user's ideal. The set of packages to purchase is the set that has a relatively high belief and also a relatively high plausibility which implies that the set of competing packages has relatively low belief. For example, if the belief in an alternative is 0.7 and plausibility is 0.6, then the belief in the competition is  $1 - 0.6 = 0.4$ . If, on the other hand, the belief is 0.5 and the plausibility is 0.9 then the belief in the competition is  $1 - 0.9 = 0.1$ . Although,

the belief in the second alternative is lower than the first, the combination of belief and plausibility is greater. This fact, plus the lower belief in the competition's ability to satisfy the decision maker makes the second alternative the more viable choice.

Most computer users would like software packages to be extremely user friendly. Possible attributes that would help accomplish this goal include: 1) pull-down menus, 2) built-in model editors, 3) output viewer, 4) on-line help, 5) automatic menu selection, 6) explicit documentation, 7) ease of debugging, 8) printing buffer, 9) computational speed, and 10) helpful execution error messages.

Let us consider a very simple example of this software selection problem whereby the attributes of importance to the decision maker/user are the documentation (Documentation) and on-line help (Help). The DM determines his highest attainable degree of satisfaction for Documentation = {inadequate, adequate, extensive} and Help = {undesirable, acceptable, desirable}. The decision maker performs pairwise comparisons of each possible Documentation and Help element, allocating 100 points to indicate his relative preference for one element over another.

Given the example above with three elements for Documentation, three pairwise comparisons would be made. The decision maker may assign the following:

Inadequate	<u>15</u>
Adequate	<u>85</u>

Adequate	<u>60</u>
Extensive	<u>40</u>

Extensive	<u>75</u>
Inadequate	<u>25</u>

to indicate that a package with adequate documentation is almost six times as important to him as a package with inadequate documentation, while software with extensive documentation is three times as important as a package with inadequate documentation. However, adequate documentation is one and one half times as important as extensive documentation. This indicates that the DM will in all likelihood compromise between a package with adequate and a package with extensive documentation, but is unlikely to accept one with inadequate documentation. This decision maker may have assigned these values because of experience with extensive documentation that although extensive is frequently too cumbersome and less useful for the occasional user than on-line help. Adequate documentation with a desirable on-line help is preferable to this decision maker. This is further supported by the decision maker's relative weights whereby desirable on-line help is more important than acceptable help and undesirable help is virtually not a consideration.

Using the DM's pairwise comparisons, the following calculations would be performed with Guilford's constant-sum method ([3],[5]). Matrix A is composed of all  $a_{ij}$  such that  $a_{ij}$  = the allocation of element j when compared to element i. For our example, Matrix A would be:

	Inadequate	Adequate	Extensive
Inadequate	-	85	75
Adequate	15	-	40
Extensive	25	60	-

Next, Matrix B is formed such that  $b_{ij} = a_{ij} / a_{ji}$ . Calculating from Matrix A for our example, Matrix B would be:

	Inadequate	Adequate	Extensive
Inadequate	1.00	5.67	3.00
Adequate	0.18	1.00	0.67
Extensive	0.33	1.50	1.00

Finally, Matrix C is formed as  $c_{ij} = b_{ij} / b_{i,j+1}$  where  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, n-1$ . Matrix C for our example is:

	Inadequate/Adequate	Adequate/Extensive
Inadequate	0.18	1.89
Adequate	0.18	1.50
Extensive	0.22	1.50
$\mu$	0.19	1.63
$\sigma$	0.001	0.05

Although all elements of a column represent the same ratio, they are not necessarily equal to one another. This is because of the inconsistencies in human judgment. A standard deviation beyond 0.05 has been shown to indicate a significant inconsistency of judgment by the decision maker [1]. It is suggested that the DM be encouraged to reevaluate his pairwise comparisons if this occurs. The column means as the average ratios of the decision maker are used to smooth out the variations. Assigning 1.00 to Extensive, Adequate =

$(1.63)(1.00) = 1.63$ ; and  $\text{Inadequate} = (0.19)(1.63) = 0.3097$ .  
 Normalizing and rounding to tenths, the relative weights of  
 the decision variables,  $d_i^{k_i}$ , are  $d_{\text{doc}}^{\text{Inadequate}} = 0.1$ ;  $d_{\text{doc}}^{\text{Adequate}}$   
 $= 0.6$ ; and  $d_{\text{doc}}^{\text{Extensive}} = 0.3$ .

The pairwise comparison process would be repeated for  
 each element,  $f_i^{k_i}$ , of the designated attributes to determine  
 the user's ideal as  $\sum_{k_i=1}^{n_i} d_i^{k_i} / f_i^{k_i}$ . Using the weights we  
 determined for the Documentation elements and assuming the  
 following relative importance weights have been determined for  
 the Help elements, the ideal could be expressed as  $\text{Ideal} =$   
 $\text{Documentation} + \text{Help}$  where: Documentation:  $0.1/\text{Inadequate} +$   
 $0.6/\text{Adequate} + 0.3/\text{Extensive}$  and Help:  $0.1/\text{Undesirable} +$   
 $0.4/\text{Acceptable} + 0.5/\text{Desirable}$ . Thus, the ideal indicates the  
 highest attainable degree of satisfaction of the decision  
 maker in compromising between the elements of specific  
 attributes.

This information is next combined into  $F$ , the set of all  
 possible combinations of different attributes' elements. These  
 are the focal elements of the combined mass. For our example,  
 we have nine focal elements such that

$F = \{ \text{Inadequate Documentation} \wedge \text{Undesirable Help},$   
 $\text{Inadequate Documentation} \wedge \text{Acceptable Help},$   
 $\text{Inadequate Documentation} \wedge \text{Desirable Help},$   
 $\text{Adequate Documentation} \wedge \text{Undesirable Help},$   
 $\text{Adequate Documentation} \wedge \text{Acceptable Help},$   
 $\text{Adequate Documentation} \wedge \text{Desirable Help},$



Extensive Documentation  $\wedge$  Undesirable Help,  
Extensive Documentation  $\wedge$  Acceptable Help,  
Extensive Documentation  $\wedge$  Desirable Help)

The company's software experts must now select packages that will closely approximate the user's ideal or specify doubt that such a match exists at this time. Contained in a database, the expert has tested all new software packages and evaluated them according to designated attributes. For our application, the DM has selected Documentation and Help. Suppose the software expert(s)' evaluations yielded:

Package 1 = 0.3/Inadequate Documentation+ 0.7/Adequate Documentation+ 0.2/Extensive Documentation+ 0.4/Undesirable Help + 0.2/Acceptable Help + 0.6/Desirable Help;

Package 2 = 0.5/Inadequate Documentation+ 0.6/Adequate Documentation + 0.3/Extensive Documentation+ 0.6/Undesirable Help + 0.4/Acceptable Help + 0.8/Desirable Help; and

Package 3 = 0.4/Inadequate Documentation+ 0.1/Adequate Documentation + 0.7/Extensive Documentation+ 0.1/Undesirable Help + 0.8/Acceptable Help + 0.6/Desirable Help.

Note that the decision maker's ideal is expressed in relative importance terms and the sum of the weights for each attribute will be 1.00. However, the expert is not attempting to evaluate each package according to a relative value for

each attribute's elements, but instead to indicate to what extent he feels the software satisfies the attribute element under consideration.

We now form the set B that associates all focal elements in Documentation as:

$$B = \{F_{DOC}^{Inadequate}, F_{DOC}^{Adequate}, F_{DOC}^{Extensive}\}$$

where

$$\begin{aligned} F_{DOC}^{Inadequate} &= .3/Package1 + .5/Package2 + .4/Package3 \\ F_{DOC}^{Adequate} &= .7/Package1 + .6/Package2 + .1/Package3 \\ F_{DOC}^{Extensive} &= .2/Package1 + .3/Package2 + .7/Package3 \end{aligned}$$

The set C associates all focal elements in Help as:

$$C = \{F_{HELP}^{Undesirable}, F_{HELP}^{Acceptable}, F_{HELP}^{Desirable}\}$$

where

$$\begin{aligned} F_{HELP}^{Undesirable} &= .4/Package1 + .6/Package2 + .1/Package3 \\ F_{HELP}^{Acceptable} &= .2/Package1 + .4/Package2 + .8/Package3 \\ F_{HELP}^{Desirable} &= .6/Package1 + .8/Package2 + .6/Package3 \end{aligned}$$

In order to determine the value represented by the intersection of different focal elements of Help and Documentation for each package as it relates to the DM's ideal, mass functions are specified for each focal element as:

$$\begin{aligned} m_1(F_{DOC}^{Inadequate}) &= 0.1 \\ m_1(F_{DOC}^{Adequate}) &= 0.6 \\ m_1(F_{DOC}^{Extensive}) &= 0.3 \\ m_2(F_{HELP}^{Undesirable}) &= 0.1 \\ m_2(F_{HELP}^{Acceptable}) &= 0.4 \\ m_2(F_{HELP}^{Desirable}) &= 0.5 \end{aligned}$$

Then the mass function for each focal element of the two attributes can be expressed as:

$$s_i = m(A_i) = \sum_{B \wedge C = A_i} m_1(B) m_2(C) / \sum_{B \wedge C = A_i} m_1(B) m_2(C)$$

where B and C represent focal elements of  $m_1$ ,  $m_2$  and  $A_i$  is the  $i^{\text{th}}$  focal element of  $m$ .

For this example,

$$s_1 = m(A_1) = m_1(F_{\text{DOC}}^{\text{Inadequate}}) m_2(F_{\text{HELP}}^{\text{Undesirable}}) = (0.1)(0.1) = 0.01$$

where  $A_1 = \text{Inadequate Documentation} \wedge \text{Undesirable Help}$

$A_2 = \text{Inadequate Documentation} \wedge \text{Acceptable Help}$

$A_3 = \text{Inadequate Documentation} \wedge \text{Desirable Help}$

$A_4 = \text{Adequate Documentation} \wedge \text{Undesirable Help}$

$A_5 = \text{Adequate Documentation} \wedge \text{Acceptable Help}$

$A_6 = \text{Adequate Documentation} \wedge \text{Desirable Help}$

$A_7 = \text{Extensive Documentation} \wedge \text{Undesirable Help}$

$A_8 = \text{Extensive Documentation} \wedge \text{Acceptable Help}$

$A_9 = \text{Extensive Documentation} \wedge \text{Desirable Help}$

and  $m(A_2) = 0.04$ ,  $m(A_3) = 0.05$ ,  $m(A_4) = 0.06$ ,  $m(A_5) = 0.24$ ,  
 $m(A_6) = 0.30$ ,  $m(A_7) = 0.03$ ,  $m(A_8) = 0.12$ , and  $m(A_9) = 0.15$ .

The least likelihood that the package will satisfy the DM is determined for each  $A_i$ ,  $i = 1, \dots, 9$ , by comparison of the attribute values for each package and the selection of the minimum. For example,  $A_1 = \text{Inadequate Documentation} \wedge \text{Undesirable Help}$  suggests the function  $\min(0.3/\text{Inadequate Documentation}, 0.4/\text{Undesirable Help}) / \text{Package1} + \min$

(0.5/Inadequate Documentation, 0.6/Undesirable Help) / Package2 + min (0.4/Inadequate Documentation, 0.1/Undesirable Help) / Package3 or  $A_1 = 0.3 / \text{Package1} + 0.5 / \text{Package2} + 0.1 / \text{Package3}$ .

The Belief in the  $j^{\text{th}}$  alternative is calculated as:

$$\text{Bel}(\text{Package}_j) = \sum_{A_i} \inf_{x \in \text{Package}_j} (1 - \mu_{A_i}(x)) m(A_i)$$

Then  $\text{Bel}(\text{Package1}) = (0.5)(0.01) + (0.6)(0.04) + (0.5)(0.05) + (0.4)(0.06) + (0.6)(0.24) + (0.4)(0.3) + (0.7)(0.03) + (0.3)(0.12) + (0.4)(0.15) = 0.459$ . Similarly,  $\text{Bel}(\text{Package2}) = 0.529$  and  $\text{Bel}(\text{Package3}) = 0.552$ . Thus, the third package has the highest degree of belief in satisfying the decision maker's ideal. Using belief alone, the ranking would be Package 3, Package 2 and Package 1.

Now the plausibility of Package<sub>j</sub> in our example is:

$$\text{Pls}(\text{Package}_j) = \sum_{A_i} A_i(\text{Package}_j) m(A_i)$$

and thus, the  $\text{Pls}(\text{Package1}) = (0.3)(0.01) + (0.2)(0.04) + (0.3)(0.05) + (0.4)(0.06) + (0.2)(0.24) + (0.6)(0.30) + (0.2)(0.03) + (0.2)(0.12) + (0.2)(0.15) = 0.338$ . Similarly,  $\text{Pls}(\text{Package2}) = 0.448$  and  $\text{Pls}(\text{Package3}) = 0.274$ .

The interval of uncertainty for each package  $j$  is  $[\text{Bel}(\text{Package}_j), \text{Pls}(\text{Package}_j)]$ . Thus, the interval of uncertainty for Package 1 is  $[0.459, 0.338]$ ; for Package 2 is  $[0.529, 0.448]$ ; and for Package 3 is  $[0.552, 0.274]$ .

As stated earlier, we recommend a maximizing of belief and plausibility be accomplished through a simple sum of

belief and plausibility. Using this ordering process and combining all evidence for any alternative, yields 0.797, 0.977, and 0.826; for Packages 1, 2, and 3, respectively. This would lead to a suggested final ordering of Package 2, Package 3, and then Package 1 based upon the user's ideal specification of attributes under consideration and the software expert's opinion of how each alternative satisfies those attributes.

#### IV. ALGORITHM

In general, the following algorithm can be applied to numerous multiattribute problems requiring a ranking of existing alternatives:

1. Define  $\{h_j \mid j=1,2,\dots,t\}$  as a set of existing alternatives;  
 $F_i$  for  $i=1,2,\dots,n$  as a list of attributes.
2. Let  $f_i^{k_i}$  denote elements of the attribute,  $F_i$ , where  
 $1 \leq k_i \leq n_i$  and  $1 \leq i \leq n$ .
3. Obtain focal elements,  $F_i^{k_i}$ , from  $\sum_j \alpha_{ij}^{k_i} / f_i^{k_i}$ , where  $\alpha_{ij}^{k_i}$  is the amount to which the value  $f_i^{k_i}$  is present in alternative  $h_j$  according to the expert;  $1 \leq k_i \leq n_i$  and  $1 \leq j \leq t$ .
4. Determine the ideal alternative as  $n$  fuzzy sets :  
 $\sum_{k_i=1}^{n_i} d_i^{k_i} / f_i^{k_i}$ ,  $1 \leq k_i \leq n_i$ , where  $d_i^{k_i}$  expresses to what degree the decision maker wants element  $f_i^{k_i}$  of  $F_i$ ;  
 $1 \leq k_i \leq n_i$  and  $1 \leq i \leq n$
5. Define  $n$  masses,  $m_i (1 \leq i \leq n)$  by  $m_i (F_i^{k_i}) = d_i^{k_i}$ .
6. Let  $m$  be defined by  $m = m_1 \oplus \dots \oplus m_n$ .

7. Determine  $\text{Bel}(h_j) = \sum_a (1 - \text{Max}_{x \in h_j} A_a(x)) m(A_a)$  and  $\text{Pls}(h_j) = \sum_a A_a(h_j) m(A_a)$  where  $A_a$  are fuzzy focal elements of  $m$ .
8. Determine  $\text{Bel}(h_j) + \text{Pls}(h_j)$ .
9. Rank order alternatives from highest to lowest value.

## V. CONCLUSION

In designing a decision making model like that which is detailed in the preceding algorithm, we must:

- 1) simplify the complex systems
- 2) incorporate subjective factors in a systematic way
- 3) pool evidence from independent sources of information, and
- 4) account for the uncertainty inherent in the complex decision making process.

It is obvious that the steps above are not independent. For example, when simplifying complex systems, many components are lumped together and therefore uncertainty builds up. This uncertainty is not only unavoidable, but in many cases is a by-product of taking correct steps to reduce complexity. Often diverse pieces of evidence are available. The features or attributes to which we have access are typically from different databases. In order to identify the closest available alternative to some simplified ideal, it is crucial to combine evidence about all of the attributes considered important to the decision maker. Thus, given a list of

possible decisions, and information about different attributes impacting the decision, we apply the Dempster-Shafer theory of combination of evidence to rank order alternatives to most likely satisfy the DM's ideal. Subjective factors are incorporated in the determination of this ideal and the evaluation of the available choices. Combining the information from these independent sources allows a reasonable response time to a complex decision.

The Dempster-Shafer based approach to the technology assessment problem presented in this paper is designed to aid in determining available package(s) best suited to a potential user's ideal specifications. A complete assessment of software packages would involve at least the ten factors mentioned at the beginning of the example.

It is clear that this method generalizes to other situations of technology assessment. The method is computationally intensive but can be shown to be significantly faster if a hierarchical structure of evidences is present.

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